AIAA JOURNAL
Vol. 32, No. 1, January 1994

Approximation in Nonhierarchic System Optimization

J. E. Renaud*
University of Notre Dame, Notre Dame, Indiana 46556
and
G. A. Gabriele†
Rensselaer Polytechnic Institute, Troy, New York 12180

This paper reports on the effectiveness of a nonhierarchic system optimization algorithm in application to complex coupled systems problems. A second-order nonhierarchic system optimization algorithm developed in earlier studies is modified in this study to provide for individual constraint/state modeling. A cumulative constraint formulation was used in previous implementation studies. The test problems in this study are each complex coupled systems. Complex coupled systems require an iterative solution strategy to evaluate system states. Nonhierarchic algorithm development is driven by these types of problems, and their study is imperative. The algorithm successfully optimizes each of the complex coupled systems. A significant reduction in the number of system analyses required for optimization is observed as compared with conventional optimization using the generalized reduced-gradient method.

I. Implementation Overview

THE nonhierarchic system optimization algorithm used in this research was initially proposed in Renaud and Gabriele.[1] The strategy provides for concurrent nonlinear subspace optimizations followed by a coordination procedure of system approximation. The system approximations are built using design information generated during the subspace optimizations. The original algorithm formulation (Renaud and Gabriele[1]) used an accumulated approximation strategy (Rasmussen[2]) for system approximation. The accumulated approximations were built about a first-order basis function of approximation.

Implementation of the algorithm as reported in Renaud and Gabriele[1] provided robust coordination in nonhierarchic system optimization studies. The algorithm was able to locate the well-documented optimums for the welded beam problem of Ragsdell and Phillips[3] and the speed reducer problem of Golinski.[4] The study also identified areas of the algorithm that could be improved. The first-order-based accumulated approximation of the objective function in the coordination procedure of system approximation was very accurate. However, the accumulated approximation of cumulative constraints in the coordination procedure of system approximation was observed to be less predictive. Changes in the active set within a cumulative constraint were not handled well by the first-order-based accumulated approximation. This condition produced cycling of the objective function value (i.e., oscillation) during nonhierarchic optimization.

To improve algorithm performance, the second-order-based coordination procedure of system approximation was developed in Renaud and Gabriele.[5] The second-order modification was aimed at improving the accuracy of cumulative constraint approximation in the coordination procedure. The second-order basis is obtained numerically using data already available in the design database. The design database stores design site information generated during the subspace optimizations. A quadratic polynomial approximation to the design is formed using the strategy of Vanderplaats.[6] A weighted least-squares solution strategy is employed to solve for the second-order terms in Vanderplaats' strategy. Exact data in the design database are more heavily weighted in the least-squares solution procedure. The resulting quadratic polynomial forms the basis function of accumulated approximation replacing the linear basis used in the original formulation. In Renaud and Gabriele[5] improved convergence is observed for the welded beam test problem. The improved convergence is attributed to the improved accuracy of cumulative constraint approximations when using second-order-based approximating functions.

Additional studies using the second-order-based coordination procedure of system approximation indicated that replacing the cumulative constraints with their component constraints may improve algorithm performance. Implementation of the second-order-based coordination procedure of system approximation was less effective in reducing cycling when applied to the Golinski speed reducer problem. The speed reducer cumulative constraints were composed of a large number of individual constraints as compared with the cumulative constraints in the welded beam test problem. With a larger number of individual constraints assigned to a cumulative constraint, it will more likely undergo a change in its active set during the coordination procedure. It is difficult to approximate these changes in the cumulative constraints. Inaccurate cumulative constraint approximations reduce algorithm performance and delay convergence.

Approximating individual constraints/states in the coordination procedure of system approximation should more accurately predict active set changes. The subspace optimization of Renaud and Gabriele[5] is modified to reflect individual constraint/state modeling in this study. One drawback to the modeling of individual constraints is the increased size of the design database. The use of individual constraint modeling or cumulative constraint modeling is ultimately a user's choice in algorithm implementation.

The proposed algorithm with individual constraint modeling is applied to complex coupled test problems in this study. In previous implementation studies the problem coupling was noncomplex [i.e., diagonal coefficient matrix in the global sensitivity equations (GSE)]. In this study the algorithm is seen to be effective in nonhierarchic optimization of the alkylation process problem of Sauer et al.[7] and the electronic packaging problem of Renaud[8] and Korngold et al.[9]

Received Aug. 18, 1992; presented as Paper 92-4826 at the AIAA/USAF/NASA/OAI Symposium on Multidisciplinary Analysis and Optimization, Cleveland, OH, Sept. 21-23, 1992; revision received April 25, 1993; accepted for publication April 27, 1993.

*Clark Equipment Assistant Professor, Department of Aerospace and Mechanical Engineering. Member AIAA.

†Associate Professor, Department of Mechanical Engineering, Aeronautical Engineering and Mechanics.

II. Algorithm Overview

This section presents an overview of the nonhierarchic system optimization algorithm proposed in Renaud and Gabriele[5] and modified in this research. The reader is referred to Fig. 1, which depicts the algorithm flowchart.

The method optimizes decomposed subspaces concurrently, followed by a coordination procedure of system approximation. In this study, optimization of a second-order system approximation problem is used as a coordination procedure for directing system convergence and resolving subspace conflicts. The second-order system approximation problem is formed using information obtained during concurrent subspace optimizations. This corresponds to existing design practice where individual design groups optimize their component designs, and tradeoffs or compromises are made based on the "whole" of the designers' previous experience. In the algorithm, information obtained during the subspace optimizations is used to represent designers' knowledge and data for respective design subspaces. The subspace optimizations serve to improve the component designs of each design group, while simultaneously building a design database used in the coordination procedure of system approximation. The design database stores state information for design vectors investigated during subspace optimizations. This information is "free" as it is generated by the local optimizer(s) during the subspace optimizations. The cost of storing the design database represents additional overhead intrinsic to approximation methods. System coupling is maintained and updated using the GSE approach as introduced by Sobieszczanski-Sobieski.[10,11] The algorithm, notation, and each module in Fig. 1 are detailed in Renaud and Gabriele.[5]

In general, the constraints and objective function can be treated as state variables in the nonhierarchic optimization problem. This type of definition for state variables has been successfully implemented by Sobieszczanski-Sobieski et al.,[12] Bloebaum et al.,[13] and Renaud and Gabriele.[1,5] The GSE can then be used to provide state sensitivities for optimization.

In this study each state is approximated individually in the coordination procedure of system approximation. In previous implementations, the cumulative constraint of Kreisselmeier and Steinhauser[14] was employed. The subspace optimization of Renaud and Gabriele[5] is modified to reflect individual constraint/state modeling as detailed in Eqs. (1–3). [Bold face characters denote vectors (e.g., $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{p}$, $\boldsymbol{g}$, $\boldsymbol{h}$). Bold face characters with subscripts denote subvectors (e.g., $\boldsymbol{x} = \{x_a, x_b, x_c\}$, $\boldsymbol{y} = \{y_a, y_b, y_c\}$, etc.). Nonbold face characters denote individual vector components (e.g., $x \in \boldsymbol{x}$ and $y \in \boldsymbol{y}$, etc.).] The equations depict the subspace optimization of subspace a for a generic three subspace system a, b, and c (see Renaud and Gabriele[5]).

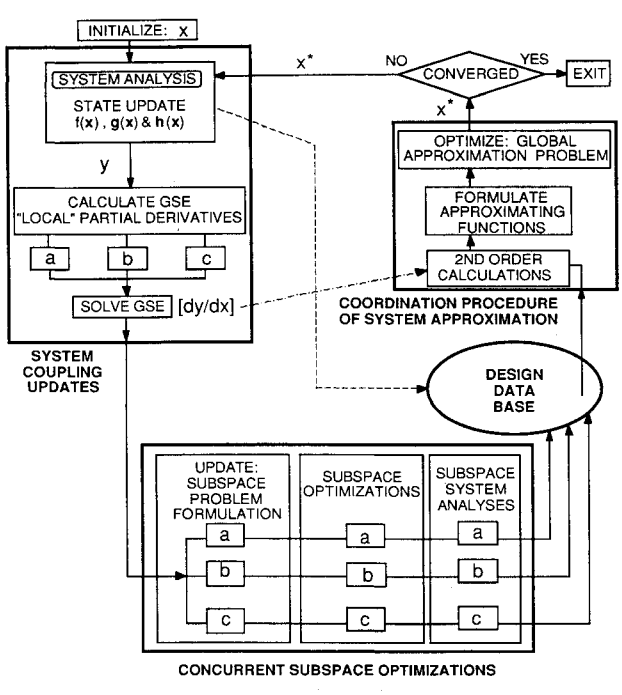

Fig. 1 Algorithm flowchart.

Equation (2) represents the output states of the local contributing analyses package used by design team "a." In general the contributing analyses are nonlinear engineering computer programs (i.e., finite element package, electronic circuit analysis, etc.). Equation (3) is a first-order approximation to nonlocal state(s) (based on GSE sensitivities) and is used as input to the local contributing analyses.

Minimize x_a:

$$f[x_a, (x_b^0, x_c^0), (y_a, y_b, y_c), p] \tag{1}$$

Subject to

$$g_a[x_a, (y_b, y_c)] \geq 0$$

$$g_b \geq 0$$

$$g_c \geq 0$$

$$\epsilon - |h_k| \geq 0 \qquad k = 1, 2, \ldots, K$$

$$(1 - ml)x_a^0 \leq x_a \leq (1 + ml)x_a^0$$

where y_a is from Eq. (2), $y_b \in \boldsymbol{y}_b$ and $y_c \in \boldsymbol{y}_c$ are from Eq. (3), $g_a \in \boldsymbol{y}_a$, $g_b \in \boldsymbol{y}_b$, $g_c \in \boldsymbol{y}_c$, h_k is from Eq. (3) if $h_k \in \boldsymbol{y}_{b,c}$ or is from Eq. (2) if $h_k \in \boldsymbol{y}_a$, ml is the local move limit ($0 \leq ml \leq 1$), and ϵ is the equality constraint tolerance,

$$y_a = y_a[x_a, (x_b^0, x_c^0), y_b, y_c] \tag{2}$$

where $y \in \boldsymbol{y}_b$ [from Eq. (3)] and $y \in \boldsymbol{y}_c$[from Eq. (3)],

$$y_i \approx y_i^0 + \left(\frac{dy_i}{dx_a}\right)^T (\Delta x_a) \tag{3}$$

where $i \in b$ or c, respectively, with $\boldsymbol{y}_b \in \boldsymbol{y}_b$ and $\boldsymbol{y}_c \in \boldsymbol{y}_c$.

III. Complex Coupled Systems

In developing a nonhierarchic decomposition and optimization algorithm, the need for robust test problems is intrinsic. This section details two complex coupled test problems developed for implementation studies. A process optimization problem developed in Colville[15] exhibits complex state coupling when decomposed into a nonhierarchic network of two subspaces. An electronic packaging problem is developed that exhibits complex coupling between electrical and thermal subsystems. The thermal energy developed in electronic components is inherently coupled to the components' own electrical performance. In electronic packaging design problems, circuit performance and thermal design (i.e., heat sink, airflow, temperature, etc.) are strongly coupled. The development of the electronic packaging test problem extends the study of nonhierarchic optimization from its current base of structures and aeroelastic control. It should be noted that the approach developed in this research is intended to be domain independent. The common attribute that motivated the development of two test problems is that each problem requires an iterative solution strategy to evaluate system states.

Alkylation Process Optimization

As part of a comparative study on nonlinear programming codes, Colville[15] details a process optimization problem. The test problem exhibits complex state coupling in that an iterative solution procedure is required to solve for both objective function and constraints. The nonhierarchic network formulation of the alkylation process optimization problem provides a robust test problem that includes complex state coupling. The problem was originally reported in Sauer et al.[7]

A simplified process flow diagram of the alkylation process optimized by Colville is given in Fig. 2. Looking at the process flow diagram, one sees that there is a reactor into which olefin feed and isobutane makeup are introduced. Fresh acid is added to catalyze the reaction, and spent acid is withdrawn. The hydrocarbon product from the reactor is fed to a fractionator, and isobutane is taken from the top and recycled back to the reactor. Alkylate product is withdrawn from the bottom of the fractionator. Some assumptions are made to keep the example simple. The olefin feed is assumed to be 100% butylene; isobutane recycle and isobutane makeup are both assumed to be 100% isobutane, and fresh acid strength is assumed to be 98% by weight.

The problem, developed at the IBM Corporation, includes three design variables and eight states. The process is constrained by 14 inequality constraints. The design variables are the controllable or "knob" variables of the process. These are the variables that can be controlled by the operator or automatic control system. The states and design variables for the alkylation process of Fig. 2 are detailed as follows:

System states:
- y_1 = profit, \$/day
- y_2 = alkylate product rate, barrels per day (bpd)
- y_3 = makeup isobutane rate, bpd
- y_4 = spent acid strength, wt%
- y_5 = motor octane number
- y_6 = external isobutane to olefin ratio
- y_7 = acid dilution factor (ADF)
- y_8 = F-4 performance no.

Design variables:
- x_1 = olefin feed rate, bpd
- x_2 = isobutane recycle rate, bpd
- x_3 = fresh acid addition rate, thousand bpd

The objective of profit maximization is modeled for minimization in this study by minimizing the negative of profit. The profit function (state y_1) is given in Eq. (4) as follows:

$$\text{Profit} = f(\boldsymbol{x}) = y_1(\boldsymbol{x}) = 0.063y_2y_5 - 5.04x_1 - 3.36y_3 - 0.035x_2 - 10.0x_3 \tag{4}$$

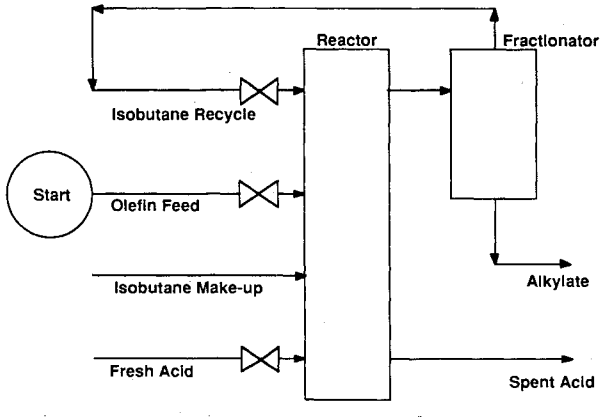

Fig. 2 Simplified flow diagram of an alkylation unit.

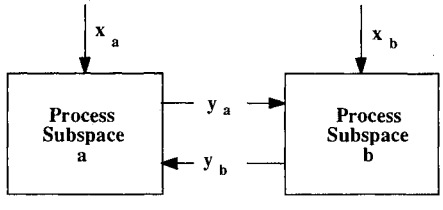

Fig. 3 Interaction between process subspaces.

The variable bounds and constraints are listed next:

Design variable bounds:
- $0.00 \le x_1 \le 2000$
- $0.00 \le x_2 \le 16{,}000$
- $0.00 \le x_3 \le 120$

State bounds (inequality constraints):
- y_1 = objective function
- $0.00 \le y_2 \le 5000$
- $0.00 \le y_3 \le 2000$
- $85.0 \le y_4 \le 93$
- $90.0 \le y_5 \le 95$
- $3.00 \le y_6 \le 12$
- $0.01 \le y_7 \le 4.0$
- $145 \le y_8 \le 162$

The states are coupled as detailed in the Fortran description of y_1 to y_8 provided in Colville.[15] To evaluate the states y_2, y_3, and y_6, an iterative solution technique is employed. Similarly the states y_4, y_5, y_7, and y_8 require an iterative solution. Decomposition into two subspaces a and b is detailed in Eqs. (5) and (6). The nonhierarchic network of process subspaces is shown in Fig. 3:

$$\{x_a\}^T = \{x_1, x_3\}$$
$$\{y_a\}^T = \{y_1, y_2, y_3, y_4\} \tag{5}$$

$$\{x_b\}^T = \{x_2\}$$
$$\{y_b\}^T = \{y_1, y_5, y_6, y_7, y_8\} \tag{6}$$

The decomposition is imposed such that the two subsystems are complexly coupled, requiring an iterative solution for state. It is assumed that each subspace (i.e., design team) has the local design expertise to evaluate the assigned local states. Nonlocal states must be approximated using the GSE sensitivities as detailed in Eq. (3):

$$\begin{vmatrix} 1 & -\frac{\partial y_1}{\partial y_2} & -\frac{\partial y_1}{\partial y_3} & 0 & -\frac{\partial y_1}{\partial y_5} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{\partial y_2}{\partial y_6} & 0 & 0 \\ 0 & -\frac{\partial y_3}{\partial y_2} & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\partial y_4}{\partial y_2} & 0 & 1 & 0 & 0 & -\frac{\partial y_4}{\partial y_7} & 0 \\ 0 & 0 & 0 & -\frac{\partial y_5}{\partial y_4} & 1 & -\frac{\partial y_5}{\partial y_6} & 0 & 0 \\ 0 & 0 & -\frac{\partial y_6}{\partial y_3} & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{\partial y_7}{\partial y_8} \\ 0 & 0 & 0 & 0 & -\frac{\partial y_8}{\partial y_5} & 0 & 0 & 1 \end{vmatrix} \tag{7}$$

The state coupling in terms of the GSE2 coefficient matrix[10] can be seen in Eq. (7). The active partial derivative inputs are listed, and the nonactive inputs are shown as zeros. The active terms are indicative of state coupling as depicted in the dependency matrix of Fig. 4. Note that the rows and columns of the dependency matrix correspond to those in the coefficient matrix of Eq. (7).

Electronic Package Optimization

In Renaud[8] and Korngold et al.[9] an electronic packaging problem is developed that exhibits complex coupling between

electrical and thermal subsystems. Figure 5 depicts a simple electronic circuit mounted on a heat sink subject to a cooling airflow. The circuit functions to divide current equally between two resistors. To achieve this function, the effective resistance of the devices must be equal. The effective resistance depends on the operating temperature of the resistors, whereas the operating temperatures depend on the effective resistances in this complex coupled problem. The temperature developed in each resistor depends on the current passing through the resistors and on the heat sink design. Coupling is complex since the current passing through each resistor is a function of the effective resistances.

The objective in this design problem is to maximize watt density for the electronic package in Fig. 5 (i.e., electronic circuit and heat sink). Watt density is equal to the total power dissipated by the circuit divided by the volume of the heat sink. The design is subject to constraints on current division and component reliability. The current division constraint requires that the current through the two resistors be equal. The reliability constraint states that the operating temperatures of the resistors should be below a threshold temperature. The design variables and states are as follows:

x_1 = heat sink width, m
x_2 = heat sink length, m
x_3 = fin length, m
x_4 = fin width, m
x_5 = nominal resistance no. 1 at temperature T^0, Ω
x_6 = temperature coefficient of electrical resistance no. 1, K^{-1}

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
y_1	●	●	●		●			
y_2		●				●		
y_3		●	●					
y_4		●		●			●	
y_5				●	●	●		
y_6			●			●		
y_7							●	●
y_8					●			●

Fig. 4 Process optimization dependency matrix.

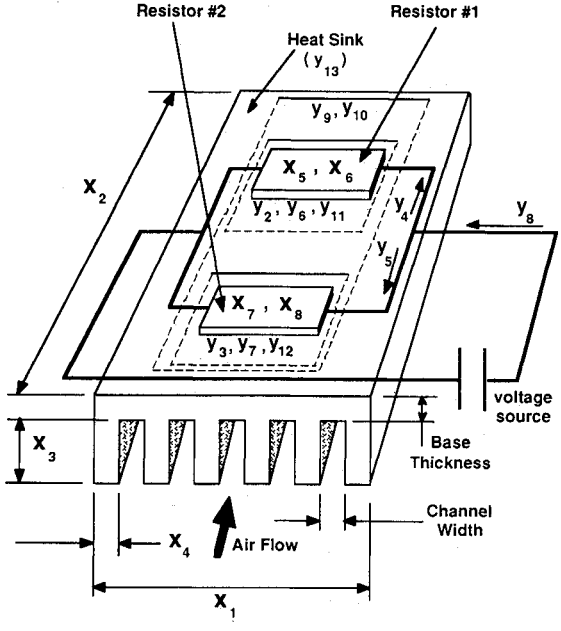

Fig. 5 Electronic package (objective function = y_1).

x_7 = nominal resistance no. 2 at temperature T^0, Ω
x_8 = temperature coefficient of electrical resistance no. 2, K^{-1}

The design variable bounds are as follows:

$$0.05 \le x_1 \le 0.15$$
$$0.05 \le x_2 \le 0.15$$
$$0.01 \le x_3 \le 0.10$$
$$0.005 \le x_4 \le 0.05$$
$$10.0 \le x_5 \le 1000.0$$
$$0.004 \le x_6 \le 0.009$$
$$10.0 \le x_7 \le 1000.0$$
$$0.004 \le x_8 \le 0.009$$

The states of the electronic package are as follows:

y_1 = negative of watt density, W/m^3
y_2 = resistance no. 1 at temperature T_1, Ω
y_3 = resistance no. 2 at temperature T_2, Ω
y_4 = current in resistor no. 1, A
y_5 = current in resistor no. 2, A
y_6 = power dissipation in resistor no. 1, W
y_7 = power dissipation in resistor no. 2, W
y_8 = total circuit current, A
y_9 = total circuit resistance, Ω
y_{10} = total circuit power, W
y_{11} = component temperature (T_1) of resistor no. 1, °C
y_{12} = component temperature (T_2) of resistor no. 2, °C
y_{13} = heat sink volume, m^3

The states are defined as follows:

$$y_1 = -y_{10}/y_{13} \tag{8}$$
$$y_2 = x_5[1.0 + x_6(y_{11} - T^0)] \tag{9}$$
$$y_3 = x_7[1.0 + x_8(y_{12} - T^0)] \tag{10}$$
$$y_4 = y_3 y_8/(y_2 + y_3) \tag{11}$$
$$y_5 = y_2 y_8/(y_2 + y_3) \tag{12}$$
$$y_6 = (y_4)^2 y_2 \tag{13}$$
$$y_7 = (y_5)^2 y_3 \tag{14}$$
$$y_8 = \text{voltage}/y_9 \tag{15}$$
$$y_9 = (1.0/y_2 + 1.0/y_3)^{-1} \tag{16}$$
$$y_{10} = (y_8)^2 y_9 \tag{17}$$
$$y_{11} = \text{implicit function}(y_6, y_7, x_1, x_2, x_3, x_4) \tag{18}$$
$$y_{12} = \text{implicit function}(y_6, y_7, x_1, x_2, x_3, x_4) \tag{19}$$
$$y_{13} = x_1 x_2 x_3 \tag{20}$$

It should be noted that the component temperatures, states y_{11} and y_{12}, are the implicit outputs of a finite difference thermal analysis of the heat sink, based on the power dissipation of the resistors and the heat sink model. The parameters in Eqs. (9), (10), and (15) of the problem are as follows: voltage = 10.0 V and $T^0 = 20.0$°C.

The heat sink parameters (heat sink material, airflow, channel width, base thickness, finite difference grid size, thermodynamic constants, etc.) and finite difference analysis package used in this research are taken from Urban.[16] Urban's heat

sink thermal analysis package evaluates component temperatures based on fixed component power dissipations. The thermal analysis package of Urban[16] is modified in this research (see Korngold et al.[9]) to include an iterative solution strategy that accounts for the component temperature impact on power dissipation in terms of component resistance changes. The relative locations of the resistors are fixed. The resistors are centered with respect to x_1 and spaced at one-third intervals with respect to x_2. The electronic package optimization problem is detailed in Eqs. (21):

Minimize:

$$y_1 \quad \text{negative of watt density} \tag{21}$$

subject to

$h_1 = y_4 - y_5 = 0.0$ branch current equality

$g_1 = 85.0 - y_{11} \geq 0.0$ component no. 1 reliability constraint

$g_2 = 85.0 - y_{12} \geq 0.0$ component no. 2 reliability constraint

Decomposition into two subspaces, thermal design (TD) and circuit design (CD), is detailed in Eqs. (22) and (23):

$$\begin{aligned} \{x_{TD}\}^T &= \{x_1, x_2, x_3, x_4\} \\ \{y_{TD}\}^T &= \{y_1, y_{11}, y_{12}, y_{13}\} \end{aligned} \tag{22}$$

$$\begin{aligned} \{x_{CD}\}^T &= \{x_5, x_6, x_7, x_8\} \\ \{y_{CD}\}^T &= \{y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}\} \end{aligned} \tag{23}$$

The nonhierarchic network of electronic packaging subspaces is shown in Fig. 6.

The state coupling in terms of the GSE2 coefficient matrix can be seen in Eq. (24). The active partial derivative inputs are listed, and the nonactive inputs are shown as zeros. The active terms are indicative of state coupling as depicted in the dependency matrix of Fig. 7. Note that the rows and columns of the dependency matrix correspond to those in the coefficient matrix of Eq. (24). The state y_1 is the objective function and must be evaluated at each subspace as noted in Eqs. (22) and (23):

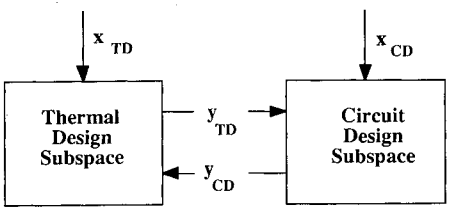

Fig. 6 Interaction between electronic package subspaces.

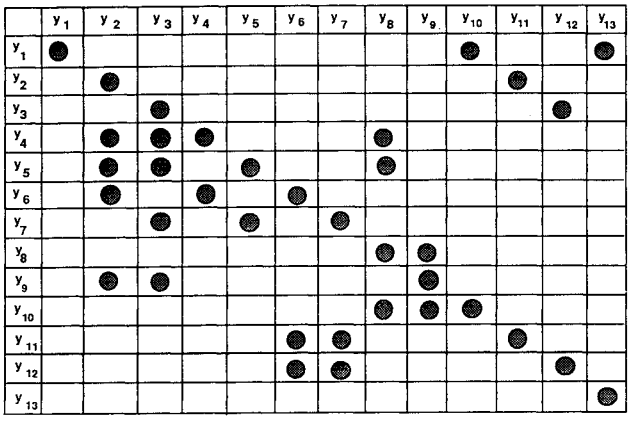

Fig. 7 Electronic package dependency matrix.

IV. Results

The generalized reduced gradient (GRG) optimizer,[17] OPT3.2, is used as the internal optimizer for both the subspace and system approximation optimizations in this research.

Alkylation Process Optimization

The nonhierarchic algorithm performed well as seen in the alkylation process optimization objective function iteration history plot in Fig. 8. Move limits of 3% on design variables

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\partial y_1}{\partial y_{10}} & 0 & 0 & -\frac{\partial y_1}{\partial y_{13}} \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\partial y_2}{\partial y_{11}} & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\partial y_3}{\partial y_{12}} & 0 \\
0 & -\frac{\partial y_4}{\partial y_2} & -\frac{\partial y_4}{\partial y_3} & 1 & 0 & 0 & 0 & -\frac{\partial y_4}{\partial y_8} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{\partial y_5}{\partial y_2} & -\frac{\partial y_5}{\partial y_3} & 0 & 1 & 0 & 0 & -\frac{\partial y_5}{\partial y_8} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{\partial y_6}{\partial y_2} & 0 & -\frac{\partial y_6}{\partial y_4} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{\partial y_7}{\partial y_3} & 0 & -\frac{\partial y_7}{\partial y_5} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{\partial y_8}{\partial y_9} & 0 & 0 & 0 & 0 \\
0 & -\frac{\partial y_9}{\partial y_2} & -\frac{\partial y_9}{\partial y_3} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\partial y_{10}}{\partial y_8} & -\frac{\partial y_{10}}{\partial y_9} & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{\partial y_{11}}{\partial y_6} & -\frac{\partial y_{11}}{\partial y_7} & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{\partial y_{12}}{\partial y_6} & -\frac{\partial y_{12}}{\partial y_7} & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \tag{24}$$

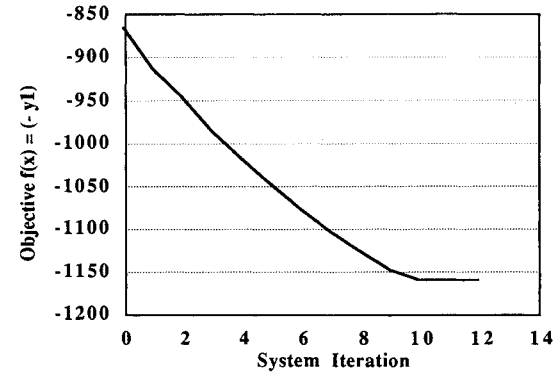

Fig. 8 Alkylation process optimization convergence history.

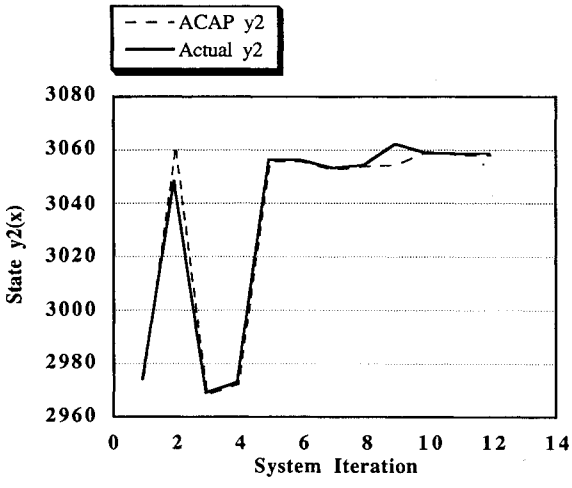

Fig. 9 Alkylation process: state $y_2(x)$ comparison plot.

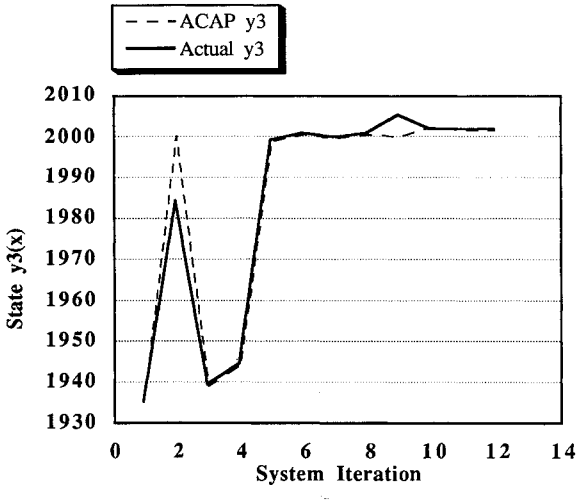

Fig. 10 Alkylation process: state $y_3(x)$ comparison plot.

were imposed for the subspace optimizations and in the coordination procedure of system approximation. The small percentage move limits were selected to account for the large magnitude design variables (i.e., $x_1 = 1700.0$ and $x_2 = 16{,}000.0$). The nonhierarchic algorithm converged to the same solution as found in a single level solution using a GRG optimizer. Table 1 details the final optimization results. No cycling is observed when using individual constraint (state) modeling since changes in the active set are easily predicted. This is considered an advantage as compared with cumulative constraint modeling.

The design database was allowed to grow to 24 design sites and thereafter to store the most recent sites visited. The subspace solution data are more heavily weighted in the least-squares solution for the second-order basis of approximation. The results reported for this section use a Rasmussen tuning parameter α of 0.01. Each state y_1 through y_8 is approximated using the second-order-based approximation detailed in Renaud and Gabriele.[5] The system approximation functions are formed about the current system design iterate.

The GRG optimization of the alkylation process optimization required 74 system analyses, whereas the nonhierarchic implementation required only 36 system analyses. The goal in multidisciplinary design optimization research is to reduce the number of actual system evaluations required for optimization. The nonhierarchic strategy exhibits a 51% reduction in the number of system analyses required for optimization of the process alkylation optimization.

The plots in Figs. 9 and 10 compare the final state approximations [ACAP $y(x)$] to the actual state values [Actual $y(x)$] for states y_2 and y_3, respectively. The final approximations are taken from the coordination procedure's approximate system optimization solution. These approximate values are compared with the actual values that are calculated as part of the system coupling updates (see Fig. 1). The plots in Figs. 9 and 10 are typical of each of the eight states and demonstrate the accuracy and utility of the second-order-based approximation of Renaud and Gabriele.[5] The use of individual state modeling is observed to be very accurate and predictive.

Table 1 Alkylation process optimization results (formulation II)

	Nonhierarchic formulation	GRG OPT3.2
Objective $f(x) = -y_1$	−1,162.36	−1,162.04
x_1	1,729.23	1,728.38
x_2	16,000.0	16,000.0
x_3	98.198	98.186
y_2	3,057.61	3,056.06
y_3	2,001.06	2,000.02
y_4	90.614	90.626
y_5	94.187	94.192
y_6	10.409	10.414
y_7	2.6175	2.6140
y_8	149.56	149.57
System analyses required	36	74

Table 2 Electronic package optimization results

	Nonhierarchic formulation	GRG OPT3.2
Objective $f(x) = y_1$	−635,961.0	−635,574.0
x_1	0.0500	0.0500
x_2	0.0500	0.0500
x_3	0.0100	0.0099
x_4	0.0050	0.0100
x_5	10.017	10.026
x_6	0.0040	0.0039
x_7	10.000	10.000
x_8	0.0040	0.0039
y_2	12.5766	12.5927
y_3	12.5805	12.5813
y_4	0.79488	0.79483
y_5	0.79512	0.79410
y_6	7.94633	7.95556
y_7	7.95376	7.93381
y_8	1.59000	1.58893
y_9	6.28927	6.29352
y_{10}	15.9000	15.8893
y_{11}	83.8306	83.9939
y_{12}	84.5125	84.5325
y_{13}	0.000025	0.000025
System analyses required	36	214

Electronic Package Optimization

The nonhierarchic algorithm with individual state modeling performed well as seen in the electronic package optimization objective function iteration history plot in Fig. 11. Move limits of 10% on design variables were imposed for the subspace optimizations and in the coordination procedure of system approximation. The nonhierarchic algorithm converged to the same solution as found in a single level solution using a GRG optimizer. Table 2 details the final optimization results. The design database was allowed to grow to 48 design sites and thereafter to store the most recent sites visited. The results reported for this section use a Rasmussen tuning parameter α of 0.001. Each state y_1 through y_{13} is approximated using the second-order-based approximation of Renaud and Gabriele.[5]

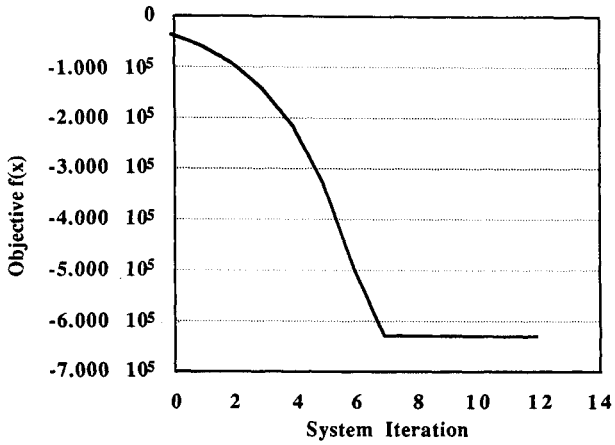

Fig. 11 Electronic package optimization convergence history.

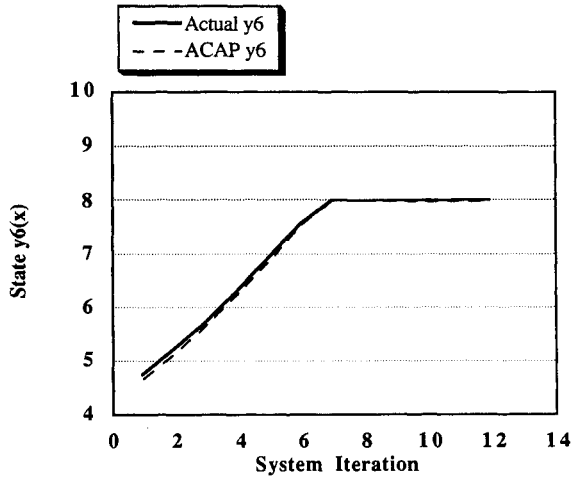

Fig. 12 Electronic package: state $y_6(x)$ comparison plot.

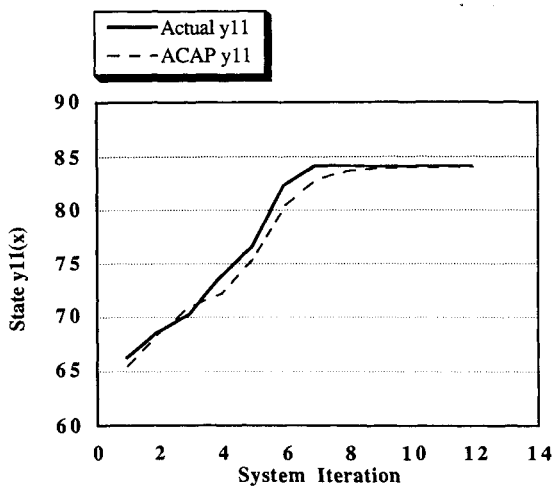

Fig. 13 Electronic package: state $y_{11}(x)$ comparison plot.

The GRG optimization of the electronic package optimization required 214 system analyses, whereas the nonhierarchic implementation required only 36 system analyses. The goal in multidisciplinary design optimization research is to reduce the number of system evaluations required for optimization. The nonhierarchic strategy exhibits an 83% reduction in the number of system analyses required for optimization of the electronic package. In addition, a small reduction in the optimal objective function value is observed.

The plots in Figs. 12 and 13 compare the final state approximations [ACAP $y(x)$] to the actual state values [Actual $y(x)$] for states y_6 and y_{11}, respectively. The final approximations are taken from the coordination procedure's approximate system optimization solution. These approximate values are compared with the actual values that are calculated as part of the system coupling updates (see Fig. 1). The plots in Figs. 12 and 13 are typical for each of the state approximations and demonstrate the accuracy and utility of the second-order-based approximation in application to individual states.

V. Conclusions

This research demonstrates the effectiveness of a second-order nonhierarchic system optimization algorithm in application to complex coupled systems problems. The alkylation process and the electronic package studied in this research are each complex coupled systems requiring an iterative solution for state. Nonhierarchic algorithm development is driven by these types of problems and their study is imperative. The nonhierarchic algorithm successfully optimizes each of the complex coupled systems. A significant reduction in the number of system analyses required for optimization is observed as compared with conventional optimization using the generalized reduced-gradient method.

Objective function cycling at convergence was observed in previous studies.[8] The cycling was attributed to inaccuracies in the approximation of cumulative constraints occurring in the algorithm's coordination procedure. In this study the use of individual constraint/state modeling in the coordination procedure of system approximation is observed to be highly predictive. As a result, the objective function convergence histories of the complex coupled systems do not exhibit cycling.

Future studies must address problem size restrictions. In this study individual state modeling and storage in the design database did not present themselves as issues. When solving larger problems, designers will need to consider the tradeoff between the benefit of increased accuracy in individual state modeling vs the data storage reductions of cumulative constraint modeling. The large reduction in the number of system analyses reported in this research does not fully compare the computational efficiencies of the two methods (i.e., nonhierarchic vs GRG). Computational work at the subsystem design level in the nonhierarchic approach is not reflected in a system analyses comparison. Recognizing the need for improved efficiencies, scientists are currently investigating two new evolutions of the algorithm. One evolution involves a new state space decomposition that uses local design databases and eliminates the subspace system analyses currently required. In addition a more efficient system approximation is proposed where local design databases are used to form second-order GSE sensitivities. A second investigation is focused on the use of artificial neural networks to build system approximations using information stored in the design database.

References

[1]Renaud, J. E., and Gabriele, G. A., "Sequential Global Approximation in Non-Hierarchic System Decomposition and Optimization," *Advances in Design Automation, Design Automation and Design Optimization,* 19th Design Automation Conference (Miami, FL), DE-Vol. 32-1, edited by G. Gabriele, American Society of Mechanical

Engineers, New York, 1991, pp. 191–200.

[2]Rasmussen, J., "Accumulated Approximation—A New Method for Structural Optimization by Iterative Improvement," *Proceedings of the Third Air Force/NASA Symposium on Recent Advances in Multidisciplinary Analysis and Optimization* (San Francisco, CA), edited by V. B. Venkayya, Flight Dynamics Directorate, Wright Laboratory, Wright-Patterson AFB, OH, 1990.

[3]Ragsdell, K. M., and Phillips, D. T., "Optimal Design of a Class of Welded Structures Using Geometric Programming," *ASME Journal of Engineering for Industry,* Series B, Vol. 98, No. 3, 1975, pp. 1021–1025.

[4]Golinski, J., "Optimal Synthesis Problems Solved by Means of Nonlinear Programming and Random Methods," *Journal of Mechanisms,* Vol. 5, Pergamon, Oxford, England, UK, 1970, pp. 287–309.

[5]Renaud, J. E., and Gabriele, G. A., "Improved Coordination in Nonhierarchic System Optimization," *AIAA Journal,* Vol. 31, No. 12, 1993, pp. 2367–2373; also AIAA Paper 92-2497, April 1992.

[6]Vanderplaats, G. N., *Numerical Optimization Techniques for Engineering Design,* McGraw-Hill, New York, 1984, pp. 211–215.

[7]Sauer, R. N., Colville, A. R., and Burwick, C. W., "Computer Points the Way to More Profits," *Hydrocarbon Processing & Petroleum Refiner,* Vol. 43, No. 2, 1964, pp. 84–93.

[8]Renaud, J. E., "Sequential Approximation in Non-Hierarchic System Decomposition and Optimization: A Multidisciplinary Design Tool," Ph.D. Thesis, Rensselaer Polytechnic Inst., Troy, NY, 1992, Chap. 3.

[9]Korngold, J., Kott, G., Renaud, J. E., and Gabriele, G. A., "Multidisciplinary Design Optimization Applied to Electronics Packaging," *Proceedings of the Fourth AIAA/Air Force/NASA/OAI Symposium on Multidisciplinary Analysis and Optimization* (Cleveland/Independence, OH), AIAA, Washington, DC, 1992, pp. 130–140 (AIAA Paper 92-4704).

[10]Sobieszczanski-Sobieski, J., "On the Sensitivity of Complex, Internally Coupled Systems," *Proceedings of the AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics, and Materials Conference* (Williamsburg, VA), AIAA, Washington, DC, 1988; also *AIAA Journal,* Vol. 28, No. 1, 1990, pp. 153–160 (originally NASA TM-100537, Jan. 1988).

[11]Sobieszczanski-Sobieski, J., "Optimization by Decomposition: A Step from Hierarchic to Non-Hierarchic Systems," Second NASA/Air Force Symposium on Recent Advances in Multidisciplinary Analysis and Optimization, NASA Conference Publication 3031, Pt. 1, Hampton, VA, Sept. 1988.

[12]Sobieszczanski-Sobieski, J., Bloebaum, C., and Hajela, P., "Sensitivity of Control Augmented Structure Obtained by a System Decomposition Method," *Proceedings of the AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics, and Materials Conference* (Williamsburg, VA), AIAA, Washington, DC, 1988 (originally NASA TM-100535, Jan. 1988).

[13]Bloebaum, C. L., Hajela, P., and Sobieski, J., "Non-Hierarchic System Decomposition in Structural Optimization," *Engineering Optimization,* Vol. 19, Gordon & Breach, London, 1992, pp. 171–186; also Third Air Force/NASA Symposium on Recent Advances in Multidisciplinary Analysis and Optimization, San Francisco, CA, Sept. 1990.

[14]Kreisselmeier, G., and Steinhauser, R., "Systematic Control Design by Optimizing a Vector Performance Index," *Proceedings of the International Federation of Active Control Symposium on Computer Aided Design of Control Systems* (Zurich, Switzerland), Aug. 1979, pp. 113–117.

[15]Colville, A. R., "A Comparative Study on Nonlinear Programming Codes," IBM New York Scientific Center Rept. 320-2949, IBM Corp., Philadelphia Scientific Center, Philadelphia, PA, June 1968, Test Problem 8, p. 31.

[16]Urban, C. T., "Computer Aided Design and Component Placement Optimization for High Frequency Switching Power Supplies," M.S. Thesis, Rennselaer Polytechnic Inst., Troy, NY, 1991, Chap. 5, pp. 89–101.

[17]Gabriele, G. A., and Beltracchi, T. J., "OPT3.2: A FORTRAN Implementation of the Generalized Reduced Gradient Method," Users Manual, Dept. of Mechanical Engineering, Aerospace Engineering and Mechanics, Rensselaer Polytechnic Inst., Troy, NY, 1988.